EXAMPLE OF A MATHEMATICS MASTERY DEPARTMENTAL WORKSHOP FOR SECONDARY SCHOOLS
INTRODUCTION

We believe that teachers are best placed to plan their own lessons, tailored to individual classes.

Rather than offering prescriptive lesson plans, Mathematics Mastery provides an online toolkit, rich with tasks and activities, which teachers can use and adapt for students. These form the building blocks for planning upcoming lessons and delivering outstanding Mathematics Mastery teaching.

In order to facilitate this, departments are asked to spend one hour per week working together to plan how to deliver Mathematics Mastery lessons. The workshops will be centred on curriculum content and, occasionally, other areas of professional development. This offers teachers the opportunity to plan lessons collaboratively and share knowledge, expertise, ideas, challenges and experience.

In order to create well planned and executed lessons, we have developed a framework for teachers to run their Departmental Workshop meetings. This encourages thinking about the key elements that create a good lesson such as differentiation, questioning, language and misconceptions. By working through the Mathematics Mastery tasks, teachers can then better identify potential problems that may occur in their classroom and will support in their delivery of effective and engaging lessons.

“I can’t stress enough how vital it is for teachers to complete the tasks before teaching them. By doing this I have found it far easier to anticipate what my students might do or where they may struggle (particularly with the open problems) so I can plan scaffolding carefully. On the occasions I haven’t done this I have ended up having very ‘teacher-led’ lessons which were not as effective.”

Emily Hudsmith, Head of Maths, Charter Academy
DEPARTMENTAL WORKSHOP

During a workshop teachers are expected to do some maths!
We believe that this forms the most important step in the lesson planning process. If teachers complete the set tasks they are more likely to come across potential problems that may arise in a lesson and appreciate the misconceptions and difficulties students may have. Furthermore, by doing the tasks and sharing ideas, all teachers will be able to gain a greater appreciation for the variety of ways in which mathematics tasks can be approached. We actively encourage teachers to take our resources and adapt them for their use.

After completing the tasks and discussing the key ideas we recommend more general discussions about teaching the unit such as:
- Use of technical language
- Use of questioning in class
- Differentiation through depth, not content
- Common misconceptions and how to address them

The workshop resources that we provide offer extra guidance on these points. There are lots of questions to stimulate discussion and an additional guidance document with a brief discussion about the tasks completed as well as answers to the problems. There are also examples of good questions, differentiation ideas and some misconceptions to get teachers thinking.

The Departmental Workshop should be seen as the first step in lesson planning. After doing the maths, discussing the tasks and other elements of the unit, each teacher will then need to spend some time planning lessons for their individual classes. The resources we provide can and should be amended where appropriate so that learning is personalised to suit the needs of the students in each class. In total, there are 3 supporting resources for each unit at Departmental Workshops:

1. WORKSHOP OVERVIEW
   These are the key mathematics tasks which teachers will complete and form the basis of workshop discussions. All questions on this should be considered and discussed.

2. QUESTIONS BY TOPIC
   This delves deeper into elements of language and questioning, differentiation and misconceptions relevant to the unit.

3. IN-DEPTH SUPPORT GUIDE
   This provides detailed, topic specific questions and ideas to stimulate conversation, if required, during the workshop. We would only recommend using this if the team are stuck and need further prompting.

Following on from this is an example of these three resources in a Year 7 unit >>
Year 7 Unit 5
Multiplication of whole numbers

1. WORKSHOP OVERVIEW

These are the key mathematics tasks which teachers will complete and form the basis of workshop discussions. All questions on this should be considered and discussed.

This workshop focuses on exploring multiplication of whole numbers, through exploring multiples, common multiples and using the column method for multiplication. In particular, bar model representations used for addition and subtraction are extended to multiplication. Division may be met as the inverse of multiplication, but division is not formally taught until Unit 7.

Do the maths

Doing the maths as a department will help you identify key teaching points, think about misconceptions and difficulties that may arise and how tasks will need to be adapted to suit your students. The resources provided are all designed to be edited for this reason.
Some questions to consider with your colleagues during the Departmental Workshop:

- What misconceptions may come up during this unit?
- How will you use manipulatives to enhance understanding when doing these tasks?
- What prior knowledge do you expect from your students? How will you find out?
- How will you try to ensure that students understand the concept of multiplication rather than learn a procedure?
- How can you encourage pupils to develop their mental calculation strategies?
- What will your students need to know to access this task?
- Is the opportunity here for extended investigations?
- What extra support might students need?
2. QUESTIONS

BY TOPIC

This delves deeper into elements of language and questioning, differentiation and misconceptions relevant to the unit.

Language & Questioning

- Can you come up with some key questions you may wish to use during this unit?

- What is the key technical terminology that you will be using in this unit? How will you ensure students understand and use this language?

- Often, it is hardest to define simple concepts. How would you define a factor? Or a multiple?

- List some words that are different ways of referring to multiplication, such as triple, or lots of. Will you try to use them all in your teaching? Why/why not?

- How can we help students to “see”/experience multiplication in different ways and not just as repeated addition?

We recommend the following links to e-learning modules in our online toolkit, which contain a wealth of information about questions and questioning techniques. These form part of our integrated CPD for Mathematics Mastery teachers.

CPD module on questioning
Planning questions for learning
NRICH article about types of questions
Differentiation

- How might you stretch students during this unit without changing the content of the work? For example, can you come up with any investigations centred on the theme of factors or multiples?

- Do you need to make any changes to the resources provided?

- Do you already have some resources for this unit that would support or challenge students?

- Can bar models be used to extend or support students? How?

Common Misconceptions

- How could the use of manipulatives be used to address misconceptions? Could they be the cause of any misconceptions?

- Students will have met multiplication tables, multiples and factors at primary school. How can you find out what they already know and what misconceptions they hold?

- “To multiply by 10 you just add on a zero” – whilst this produces the correct answers in this integers unit, how will you address this misconception now, rather than wait until the decimals unit?

- A student has written:

  \[
  \begin{array}{c}
  \times 3 \\
  36 \\
  \hline
  48 \\
  \hline
  1
  \end{array}
  \]

Can you explain what they may have done wrong? Can you be sure? How?
3. IN-DEPTH SUPPORT GUIDE

This provides detailed, topic specific questions and ideas to stimulate conversation, if required, during the workshop. We would only recommend using this if the team are stuck and need further prompting.
1. WORKSHOP OVERVIEW

**Swap the digits**
This task serves two key purposes: firstly, it encourages students to work with counters and manipulatives and to develop their understanding of multiplication. In addition, it builds familiarity with multiplication pairs, for example, through the use of a multiplication grid.

This task can be used to check prior knowledge of multiplication and multiples. Whilst carrying out the task, you will be able to check to see what methods students employ and how well they recognise multiplication pairs.

**Answers:**
4x2=8, 5x7=35, 3x6=18, 6x8=48, 4x9=36, 3x4=12, 6x9=54, 12x5=60, 21x8=168, 31x21=651

**Find the missing digits**
It is likely that students have been taught different methods of multiplication in primary school. We would encourage discussion of the other methods and in particular their strengths and weaknesses. It is vital that students are exposed to and encouraged to become familiar with how they work and why they work.

This task also encourages students to recall and use multiplication facts. The decision of when to introduce long multiplication is left up to the teacher.

**Answers:**
343x2=686, 436x4=1744, 514x7=3598, 47x13=511, 38x72=2736, e.g. 93x30=2790 or 90x31=2790.

**Equal Value**
This task allows students to practise written multiplication skills and also to develop their conceptual understanding through the use of bar models. It may be easier to start with smaller numbers (and the lesson resources do start with smaller numbers) and ensure students are familiar with and understand the bar model application.

**Answers:**
252: 12x21, 63x4, 3x84, 7x36, 14x18.
2. QUESTIONS BY TOPIC

Language & Questioning

There are a variety of types of questions and they serve different purposes. Questions can:
- Offer a starting point
- Serve as a prompt
- Encourage conceptual thinking
- Check understanding
- Encourage justification
- Encourage reflection

Some suggested questions for this unit:
What is the next multiple? How do you know?
What operation is being performed to get a multiple of a number?
What are common multiples?
How does knowing multiples help?
What is the lowest common multiple? Is this always the product of the two?
What number facts are you using?
What is the question asking for?

Bar Questions

This is a really nice task to draw together several skills. In writing problems that can be represented by the given models, students demonstrate their understanding of multiplication and modelling. In addition, they have the chance to develop their technical language skills.

You may find it helpful to demonstrate use of bar models before moving onto this task. There are suggested problems in the lesson resource pack.

Answers:
There are three simple problems that could be used, but there are numerous others.
1. Purple bar: Tom buys 5 apples. Each apple costs 23p. How much is this altogether?
2. Blue bar: James has 16 pencils. George has five times as many pencils as James. How many pencils does George have? How many pencils do they have altogether?
3. Green bar: Alex has £32. After spending £7, she now has five times as much money as Betty. How much money does Betty have?
Differentiation

Ideas for stretching students during this unit:
- Create bar models when problem solving
- Create more than one model for a given question
- Explain the lowest common multiple to peers
- Investigations and worded problems involving factors and multiples
- Extend use of Venn diagrams in other contexts

Ideas for supporting students during this unit:
- Use of pictorial representations
- Use of concrete manipulatives for Venn diagrams
- Use of colours:

\[
\begin{array}{c}
23 \\
\times 45 \\
\hline
115 \\
920 \\
\hline
1035
\end{array}
\]

- Remove additional complications from questions such as in area questions where units need to be converted first
- Offer templates for column method

Misconceptions

Some possible misconceptions:
- Divisibility criteria:
  - E.g., thinking a number ending in a 9 is divisible by 9
- Thinking 1 is a prime number
- Confusing HCF and LCM as well as factors and multiples in general
- Understanding (and remembering) the need for zeros at each level when using the column method
- Thinking that \(0 \times a = a\) (in part, confusion with addition)
- To multiply by 10 you just add a zero
- Errors in the column method (for example mixing it up with the addition algorithm)
- Believing that multiplication gives a bigger answer (whilst not dealing with decimals here, the counterexample of \(a \times 0 = 0\) should be reinforced)